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III. RELATING TO THE GRAPH OF $Y = f(X)$ FOR COMPLEX VARIABLES.

By E. L. REES, University of Kentucky.

The following is to suggest a slight modification of the geometric representation of $Y = f(X)$, (X and Y both complex) given by Professor Frumveller in the November, 1917, number of the MONTHLY.

Let $X = x + iy$ and $Y = u + iv$. Letting u be represented by perpendiculars to the plane of X we get a surface which pictures the variation of the real part of Y. On this surface draw the contours for $v = v_1$, v_2, etc., the consecutive v's differing by a constant. These contours enable us to visualize the variation of v, so that we have pictured the variation of both u and v and, therefore, of Y, all on one surface.

Let the surface be cut by planes parallel to the X-plane, the planes of all of the consecutive pairs being the same distance apart. Project these curves of intersection and the contours on the X-plane. The curves into which these curves project will be the images of the grating of lines parallel to the u and v axes in the Y-plane. We thus establish a simple relation between this scheme and the usual scheme of representation of functional correspondence in complex variable theory.

For the linear function the surface is a plane and the contours are lines in this plane perpendicular to its xy-trace. If the coefficients are real the plane is parallel to the y-axis.

For the quadratic function the surface is a hyperbolic paraboloid and the contours are the intersections of this surface and hyperbolic cylinders.

For $Y = \log X$ the locus is a funnel-shaped surface generated by revolving $u = \log x$ about the u-axis and the contours are the meridians of this surface.

The projections of these contours and the intersections of the surfaces by the parallel planes on the X-plane give the familiar curves associated with these functions.

BUREAU OF INFORMATION.

JAMES BYRNIE SHAW, CHAIRMAN, University of Illinois, Urbana.

The Bureau of Information was established to furnish information to any one about any mathematical subject or question. So far as possible the committee will answer questions of the following character:

1. Definitions of terms.
2. Brief explanations of recent developments in mathematics.
3. Inquiries as to theorems.
4. Inquiries as to articles upon special topics.
5. Bibliographical references.
6. Historical questions.
7. Inquiries for book reviews.

8. Indication of libraries which will lend books.
9. Information as to graduate courses, fellowships, etc.
10. Inquiries as to societies, prizes, etc.

Whenever an inquiry seems to be of sufficient general interest to warrant publication, an article relative to it will appear in the MONTHLY.

The following question is of general interest and the reply is printed below:

"It is well-known that the equilateral triangle, the square and the hexagon, are the only regular figures that will perfectly fill a plane surface. What are the closed forms (polyhedra) that will completely fill space?"

REPLY BY ARNOLD EMCH, University of Illinois.

The cellular subdivision of space is one of the important problems of analysis situs and has been the subject of a great number of investigations. It comprises the theory of crystalline forms and structure, the regular division of the surface of a sphere, etc.

In this connection two representative treatises may be mentioned: *Mathematical crystallography*[1] by H. Hilton and *Vielecke und Vielflache. Theorie und Geschichte*[2] by Dr. M. Brückner, where certain aspects of the problems in question are thoroughly treated. In Brückner's book also numerous historical and other references may be found.

In the plane there are three cases of regular polygons by which a certain portion of the plane may be packed uniformly, namely, the triangle, the square, and the hexagon. In space of three dimensions there are equal cubes, belonging to the class of regular polyhedrons, by which a certain portion of space may be uniformly filled.

If we do not limit ourselves to regular polyhedrons then there are innumerable possibilities of subdividing a certain portion of space by a set of equal polyhedrons of semi-regular and irregular types. A few characteristic cases shall be given.

In the first place space may be subdivided in an infinite number of ways by rhombohedrons or parallelepipedons, whose faces are parallelograms which, in sets of two, are equal.

The subdivision itself is obtained as follows: Take three distinct concurrent, non-coplanar axes which are supposed to be indefinitely extended. Denote the axes by X, Y, Z, and the origin by O. From O measure off on these axes (on each axis with a definite unit-length) equal distances, so that on each axis the scale of real integers is represented. Through the division points on the X-axis draw planes parallel to the YZ-plane; through those of the Y-axis planes parallel to the ZX-plane; through those of the Z-axis planes parallel to the XY-plane. These three sets of planes divide space into a uniform set of equal parallelepipedons, whose vertices form a space-lattice. There is an infinite number of subdivisions possible where the faces of the parallelepipedon are equal rhombs. In this particular case the parallelepipedon is called a rhombohedron. Of the class of rhombohedrons one is of particular importance as will appear from the construction of the rhombus-dodecahedron, Figure 1, which is another example of a semi-regular polyhedron by whose repetition a certain portion of space may be uniformly packed.

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[1] The Clarendon Press, Oxford, 1903.

[2] B. G. Teubner, Leipzig.

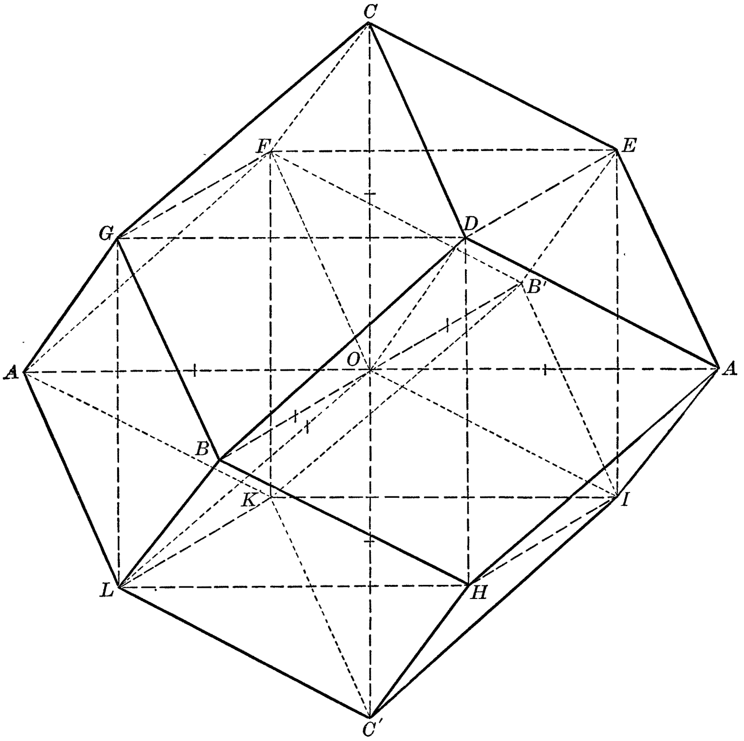

FIG. 1.

Draw three equal, mutually perpendicular axes AA', BB', CC', intersecting at O, and so that $OA = OA'$, $OB = OB'$, $OC = OC'$. Thus $AA'BB'CC'$ are the vertices of an octahedron. Now draw the perpendicular bisecting planes of the segments OA, OA', OB, OB', OC, OC'; they bound a cube $DEFGHIKL$. Join the vertices of the octahedron to those of the cube, as indicated in the figure. The joins thus obtained form the edges of the rhombus-dodecahedron, whose faces are equal rhombs with angles θ and ϕ, for which $\tan \theta/2 = \sqrt{2}$, or $\theta/2 = 54^\circ\ 42' +$, and $\phi = 180 - \theta$. The planes of two adjacent rhombs include an angle of 120°. The rhombus-dodecahedron can be divided into the four equal rhombohedrons $ADOIHBLC'$, $DCFOBGA'L'$, $ADOIDCEB'$, $OIB'FLC'KA'$.

If we make an isometric projection of the figure, *i. e.*, if we assume in the figure OA, OB, OC at angular intervals of 120°, then OD which lies in one of the principal diagonals of the cube will merely appear as a point O, Figure 2. Likewise with the edges AI, HC', BL, GA', CF, EB', which are equal and parallel to *DO*. The whole rhombus-dodecahedron appears as a regular hexagon with its radial lines, as represented by the internal hexagon around *O* in Figure 2. Without entering into further details, this shows that the rhombus-dodecahedron may be slipped into the space of a regular hexagonal prism. We can now add six other equal rhombus-dodecahedrons 1, 2, 3, 4, 5, 6, as shown in the figure, which fit in closely around *O*. Other dodecahedrons of the same kind may be fitted in horizontally and vertically, so as to pack uniformly a certain portion of space, which may be made as large as we please. If as shown in Figure 2, we fit in rhombohedrons *a, b, c, d, e, f* of the kind of which the dodecahedron is composed, *a regular hexagonal prismatic space may be packed uniformly by rhombus-dodecahedrons and rhombohedrons, or by rhombohedrons of the special form alone.*

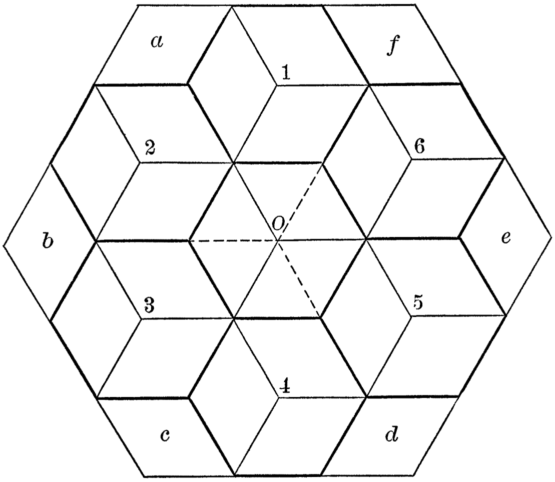

FIG. 2.

As a last example of cellular division of space the well-known case of the honey-bees' cell may be mentioned. Geometrically the cell is obtained as follows: Take three faces of a cube adjacent at one of its vertices. Through the six "open" edges of the three faces pass planes parallel to the principal diagonal of the cube passing through the common vertex of the three faces. These six planes together with the three square faces of the cube enclose essentially the form of the bees' cell.

Further references on this extensive subject may be found in the treatises mentioned above.